

Using Clifford/Geometric Algebra in Robotics

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Abstract

The aim of this lecture will be to provide the basics of Geometric Algebra (GA), beginning with its instantiation in 3d Euclidean Space and an indication that much of its power lies in the way it is able to deal with rotations via elements of the algebra we call *rotors*. To illustrate this we will look at the various methods of describing rotations, some with the minimal 3 degrees of freedom (e.g. Lie algebras, quaternions, axis-angle parameters, Euler angles) and some with more than 3 degrees of freedom which therefore require the imposition of constraints (e.g. rotation matrices, direction cosines). As much of robotics is dealing with the movements of linked rigid bodies, it is clear that a good description of rotations, translations and relative rotations is essential. We will review some of the advantages of using GA in some particular problems in robotics.

While a 3d geometric algebra provides many advantages over other systems, it is still the case that rotations and translations are fundamentally different operations in the algebra. This fact has long been a source of dissatisfaction with researchers and we will review some of the ways in which people have sought to get around this (dual quaternions, approximating a translation as a rotation with the axis at infinity etc.). In GA there is a beautiful solution to this problem – if we move to a higher dimensional space, in this case 5d, and postulate a mapping from points in Euclidean space to ‘null points’ in the 5d space, we have some astonishing simplifications. The properties of this *conformal* space will be reviewed (and the reasons for calling it conformal will be explained) – in particular, we will show that lines, planes, circles and spheres are all simple elements of our 5d algebra (not equations) and rotations, translations, inversions and dilations, are all the same type of operator, *a rotor*. This means that if we work in this conformal space we are able to simplify many complicated procedures. One particularly beautiful illustration of this is in interpolation. It has long been known that in order to interpolate rotations correctly, one must interpolate in a particular way; this is often done via quaternions and is widely used in both robotics and computer graphics. In a 3d GA, interpolating between rotors takes a very intuitive and straightforward form – since in 5d translations are also rotors, we are able to interpolate in precisely the same fashion. We will give examples of interpolating rotations and translations with applications in robotic path planning and computer graphics. There are other, rather surprising, applications of these techniques in the field of structures (beam buckling)! It is an amazing fact that once we have set out our toolkit for manipulating objects in the conformal space, it requires a few minor changes to represent these same manipulations in other non-Euclidean spaces (e.g spherical and hyperbolic). We will therefore have the mathematical formalism that is required to solve the problems of robotic manipulation in a complex non-Euclidean Universe (should we wish to)!

In summary, this lecture will attempt to provide the motivation and the toolkit for looking at the problems of manipulating rigid body transformations using the formalism of geometric algebra. The lecture which follows will concentrate on applying many of these techniques in the field of robotic vision.