LIE GROUPS AND LIE ALGEBRAS IN ROBOTICS

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Abstract

In this lecture the group of rigid body motions is introduced via its representation on standard three dimensional Euclidian space. The relevance for robotics is that the links of a robot are usually modelled as rigid bodies. Moreover the payload of a robot is also usually a rigid body and hence much of robotics is concerned with understanding rigid transformations and sequences of these transformations. Chasles's theorem is presented, that is: a general rigid body motion is a screw motion, a rotation about a line in space followed by a translation along the line.

The lower Reuleaux pairs are introduced. These are essentially surfaces which are invariant under some subgroup of rigid body motions. Such a surface can be the matting surface for a mechanical joint. In this way the basic mechanical joints used in robots can be classified. These surfaces turn out to have other applications in robotics. In robot gripping they are exactly the surfaces that cannot be immobilised using frictionless fingers. In robot vision the symmetries of these surfaces are motions which cannot be detected.

Next Lie algebras are introduced. The 1-parameter rigid motions about a joint are considered. The correspondence between elements of the Lie algebra and 1-degree-of-freedom joints is given. The exponential map from the Lie algebra to the group is defined and used to describe the forward kinematics of a serial robot using the product of exponentials formula. The Rodrigues formula for the exponential map is derived using a set of mutually annihilating idempotents.

The derivative of the exponential map is explored. In particular the Jacobian matrix for the forward kinematics is derived. More general results are also derived and these are used to show how the inverse kinematics problem can be cast as a set of ordinary differential equations.

Keywords: Clifford algebra, Lie algebras, Lie groups, rigid body motions, robotics.