

The chapters in this volume were presented at the July 2000 NATO Advanced Study Institute on *Twentieth Century Harmonic Analysis – a Celebration*. The conference was held at the beautiful Il Ciocco resort near Lucca, in the glorious Tuscany region of northern Italy. Once again we gathered at this idyllic spot to explore and extend the reciprocity between mathematics and engineering. The dynamic interaction between world-renowned scientists from the usually disparate communities of pure mathematicians and applied scientists, which occurred at our 1989, 1991, 1992, and 1998 ASI's, continued at this meeting.

Almost exactly one century ago harmonic analysis entered a (still continuing) golden age, with the emergence of many great masters throughout Europe. *Some* of these illustrious names were: Hardy, Littlewood, Landau, van der Corput, Hadamard, de la Vallée-Poussin, Tchebychev, Bernstein, Markov, Fejér, Riesz, Pólya, Szegő, etc. They created a wealth of profound analytic methods which were so successfully exploited and further developed by succeeding generations. This flourishing of concrete harmonic analysis is, today, as lively as ever, with such contemporary great analysts as ASI lecturers and authors Havin, Kahane, Shapiro, Weiss, Salem Prize winner Körner, and Nobel Laureate Hauptman.

In addition to its own ongoing internal development and its basic role in other areas of mathematics (number theory, differential equations, probability, statistics), physics and chemistry (visible light and infrared optics, crystallography, wave phenomena), financial analysis (time series), medicine (tomography, brain and heart wave analysis), and biological signal processing, harmonic analysis has made fundamental contributions to essentially all twentieth century technology-based human endeavors. This includes telephone, radio, television, radar, sonar, satellite communications, medical imaging, the internet, and multimedia. This ubiquitous nature of the subject is touched upon by many of the authors.

Thus, the ASI and this volume are intended not only to promote the infusion of new mathematical tools into applied harmonic analysis, but also to fuel the development of applied mathematics by providing opportunities for young engineers, mathematicians, and other scientists to learn more about problem areas in today's technology that might benefit from new mathematical insights.

Many of the world's harmonic analysis experts were principal speakers at the ASI, and their chapters appear in this volume. These renowned scientists address their talks and chapters to an audience which consists of a broad spectrum of pure and applied mathematicians, as well as a diverse group of engineers and scientists. Thus, the reader has the opportunity to learn or reinforce fundamental concepts from the individuals who have played a major role in the ongoing flourishing of harmonic analysis, and to see them discuss in accessible terms their profound contributions and ideas for future research.

Victor Havin begins this book by clearly describing, both heuristically and precisely, how one should view "smallness" in the context of the uncertainty principle. His chapter gives the reader a fundamental understanding of one of the major themes of classical Fourier analysis, and also discusses important applications of the uncertainty principle to physics and engineering.

Harold Shapiro beautifully shows how operator theory, involving genuinely infinite-dimensional constructions, has profound applications to concrete classical harmonic analysis problems. A wonderful case in point is D. Sarason's recognition of the fundamental role played by commutativity in understanding (and getting radically new proofs of) classical interpolation and moment problems, like those usually associated with the names of Pick-Nevalinna, Caratheodory, and Schur.

Jean-Pierre Kahane offers clear and concise insight into the interactions between Baire's category theorem, Lebesgue's measure theory, and trigonometric series.

A beautiful exposition of the current state of the art in Gabor theory is presented by Guido Janssen. He details the key role played by the Gabor frame operator associated with the set of elementary signals being used in the expansion of a given signal. Working in the time, frequency, time-frequency, and Zak

transform domains, he addresses the basic problems of whether there is a Gabor frame and how to compute the dual frame.

While describing the many fascinating twists and turns that can occur when rearranging orthogonal series, Tom Körner clarifies the surprising phenomenon that wavelet expansions continue to work well under decreasing rearrangements whereas Fourier series do not. His focus is on the work of Olevskii, Tao, and others in this area.

Stephane Jaffard clearly describes the interplay between function spaces, wavelet expansions, and multifractal analysis. For example, he shows how refinements of the numerical techniques introduced to compute turbulence spectra have led to the introduction of new function spaces, which turn out to be the right setting to determine the fractal dimensions of graphs, and offer natural extensions of the Besov spaces to negative  $p$ 's.

Richard Tolimieri and Myoung An offer significant insight into the emergence at IBM of Fourier transform algorithms, a critical step in advancing the widespread use of digital computers in scientific and technological applications. They show how early efforts focused on reducing “expensive” multiplications at the cost of increasing additions, and how the recent importance of FPGA's and reconfigurable hardware has renewed the need to reduce multiplication counts. Throughout they weave in the interactions between these fascinating computational techniques and harmonic analysis.

Herb Hauptman describes an application of harmonic analysis which is yielding profound medical benefits daily, and for which he won the 1985 Nobel Prize in chemistry. Namely, he shows how the known atomicity of crystal structures and the redundancy of the observed magnitudes of the normalized structure factors of the X-ray diffraction pattern render the classical phase problem of X-ray crystallography solvable. He goes on to describe his new “Shake-and-Bake” algorithm, a completely automatic solution of the phase problem for structures containing as many as 1000 atoms when data are available to atomic resolution.

By combining the concept of frames with the Zak transform, Josh Zeevi elucidates the use of localized bases or frames in the representation, processing, compression and transmission of speech, images and other natural nonstationary signals.

Babar Saffari offers a fascinating chapter on a fascinating subject: extremal problems involving polynomials, trigonometric polynomials, and exponential sums. The depth and beauty of this “elementary” subject come alive in his exposition.

By considering function algebras, formal power series, and operator algebras, including a quantitative treatment of the Wiener-Pitt-Sreider phenomenon for convolution measure algebras on locally compact abelian groups, Nikolai Nikolski gives an in-depth survey of recent results on the phenomenon of the “invisible spectrum” for Banach algebras.

As Hugh Montgomery makes clear, a wide variety of questions of harmonic analysis arise naturally in various contexts of analytic number theory. In his chapter, Hugh gives a clear exposition of a number of examples of this type.

By focusing on two major and central mathematical areas, the radar ambiguity function and radar waveform design, Bill Moran beautifully brings forth the intimate relationship between harmonic analysis and signal processing. It is difficult to imagine a more perfect exposition for the mathematician interested in real-world signal design and analysis.

In the past 20 years or so, wavelets have emerged as one of the central topics in both pure and applied harmonic analysis. Guido Weiss and Edward Wilson present a broad overview of many aspects of the underlying mathematical theory. A highlight is their recently obtained characterization of “all” wavelets – an

important new result.

The concluding chapter consists of several lesser known (as compared, for example, to the Riemann hypothesis) but certainly worthwhile unsolved problems and conjectures of current appeal in harmonic analysis, to challenge the interested twenty-first century reader.

Numerous giants in the field, including about half of the lecturers and authors, have recently retired or will do so in the near future. A major purpose of the ASI was to afford them the opportunity to join together to share their profound wisdom with the many future stars of pure and applied harmonic analysis. A second purpose was to produce this book for current and future generations, highlighting their thoughts and insights.

The cooperation of many individuals and organizations was required in order to make the conference the success that it was. First and foremost I wish to thank NATO, and especially Dr. F. Pedrazzini and his staff, for the initial grant and subsequent help. Financial support was also received from the Air Force Office of Scientific Research (Dr. Jon Sjogren), the European office of the Office of Naval Research (Dr. Igor Vodyanoy), the Raytheon Company (Dr. Philip W. Cheney, Chief Scientist), the National Science Foundation, the Australian Research Council, Philips Research Laboratories, the European Mathematical Society, the University of Massachusetts at Boston, and Prometheus Inc. This additional support is gratefully acknowledged.

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*Jim Byrnes*, Newport, Rhode Island