

SPECTRUM SLIDING ANALYSIS

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Abstract

The Spectrum Sliding Analysis (SSA) is a dynamic spectrum analysis in which the next analysis interval differs from the previous one by including the next signal sample and excluding the first one from the previous analysis interval. Such a spectrum analysis is necessary for time-frequency localization of analyzed signals with given peculiarities. Using the well-known Fast Fourier Transform (FFT) towards this aim is not effective. Recursive algorithms which use only one complex multiplication for computing one spectrum sample during each analysis interval are more effective. The author improved one algorithm so that it is possible to use only one complex multiplication for computing two, four, and even eight (for complex signals) spectrum samples simultaneously. Problems of realization and application of the spectrum sliding analysis are also considered in the chapter.

keywords: spectrum sliding analysis, dynamic spectrum analysis, recursive algorithms, sliding spectrum algorithms & analyzers, instant spectrum algorithms & analyzers, algorithms for instantaneous spectrum digital analyzers, multichannel filter

1. Introduction

Depending on the form of signal presentation, SSA may be implemented by analog, digital, or discrete-analog spectrum analyzers. Narrow band analog filters can be used to implement SSA at some points of the frequency's working band. However, such analyzers are usually designed to analyze power spectrums and are not capable of analyzing phase spectrum and complex spectrum Cartesian constituents, which restricts their application [7]. The spectrum analysis discrete-analog method uses the discrete signal formed by sampling the signal to be analyzed with an uncrossed pulse sequence, the magnitudes of which are altered in accordance with the sample values. This type of analyzer permits spectrum analysis with quality and quantity compatibility conditions, satisfying the information completeness condition

[7], and it may be adapted for SSA [8, 9]. Spectrum digital analyzers [1] are the most popular. There are two SSA spectrum methods: in one, the origin of the reference function is matched with the interval analysis origin. This approach computes the sliding spectrum [10]. In some applications (e.g., speech recognition) there is no need for such matching. When this is the case the computation is called the instant spectrum [10]. Algorithms and devices implementing the two methods are considered in the following sections.

2. Sliding Spectrum Algorithms and Devices

The discrete sliding spectrum is given by the formula:

$$F_q(p) = \frac{1}{N} \sum_{k=q+N-1}^q f(k) W_N^{-p(k-(q-N+1))}, \quad p \in \overline{0, P-1}, \quad q = 0, 1, 2, \dots, \quad (1)$$

where q is an analysis interval index, k is a signal sample index in the limits of the sliding window $k \in \overline{q-N+1, q}$, N is the size of the processing extract, $F_q(p)$ is the complex spectrum sample at the frequency $p\Delta\omega$ at the instant $q\Delta t$, $f(k)$ is the analyzed signal sample value at the instant $k\Delta t$ and W_N^{-pk} is the complex reference coefficient symbolic representation $\exp\left(-j\frac{2\pi}{N}pk\right)$. Δt and $\Delta\omega$ are the discrete time and frequency intervals, determined by the information completeness condition [7].

Direct computation of the functional (1) requires NP complex multiplications and $(N-1)P$ additions. FFT [11] reduces this number by $\frac{N}{\log_2 N}$. Further reduction is possible. The formula:

$$F_q(p) = \left(F_{q-1}(p) + \frac{1}{N} (f(q) - f(q-N)) \right) W_N^p, \quad p \in \overline{0, P-1}, \quad q = 0, 1, 2, \dots \quad (2)$$

leads to a recursive algorithm [12] computing the sliding spectrum samples. The computer requirements (number of complex multiplications) are considerably lower using (2) rather than (1). In (1), N complex multiplications are computed for each spectrum sample ($\log_2 N$ if the FFT is used). Using (2) the same computation requires only one complex multiplication. This advantage has led to many sliding spectrum analyzers implementing (2).

The spectrum analyzer functional diagram implementing the algorithm (2) is shown in Fig. 1. The following symbols are used in the diagram:

ADC is the analog-digital converter, DR is the delay register, SU is the subtraction unit, AD is the adder, WU is the weighting unit, MU is the multiplier, RAM is the random-access memory; $f(t)$ is the analog input signal; f_q are digital readouts of the signal $f(t)$; f_{q-N} are the readouts f_q delayed by N steps; Δf_q are the readouts of the difference signal $f_q - f_{q-N}$; W_N^p are the readouts of the weighting function $\exp\left(j\frac{2\pi}{N}p\right)$; $F_q(p)$, $F_{q-1}(p)$ are the sliding spectrum readouts in the current and the preceding steps, respectively.

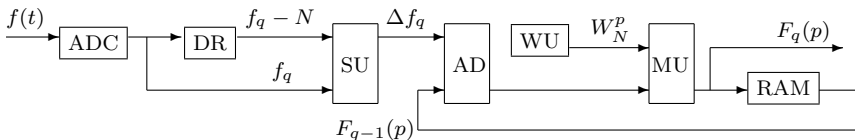


Figure 1. The spectrum analyzer functional diagram (2)

A detailed description of the characteristics of the analyzers [13-15], implementing (2), are given in [16]. Computational stability using algorithm (2) is achieved by the appropriate rounding of twiddle factor constituents having factor modulus not exceeding unity [17]. The constituents word size should be half of the sliding steps number binary logarithm [18]. For an unlimited number of sliding steps it is possible to use the technical decision described in [19]. While using algorithm (2) the reference function is always matched with the analysis interval origin, thereby ensuring a matched filtering [20-22].

3. Instant Spectrum Algorithms and Devices

The instant spectrum

$$F_q(p) = \frac{1}{N} \sum_{k=q-N+1}^q f(k) W_N^{-pk}, \quad p \in \overline{0, P-q}, \quad q = 0, 1, 2, \dots \quad (3)$$

can be implemented by the following simple recursive algorithm, in [23, 24].

$$F_q(p) = F_{q-1}(p) + \frac{1}{N} \left(f(q) - f(q-N) \right) W_N^{-pq}, \quad p \in \overline{0, P-1}, \quad q = 0, 1, 2, \dots \quad (4)$$

This algorithm is always stable and the twiddle factors W_N^{-pq} word size

is the same as the number of signal samples. Moreover, it is possible to achieve matching with an additional multiplier and a complex conjugate device [10]. In [25, 26] devices that implement this algorithm using the conveyer mode are described.

The spectrum analyzer functional diagram implementing algorithm (4) is shown in Fig. 2, where $\Delta F_q(p)$ are the increments of the spectrum readouts $\left(\frac{1}{N}\right) \Delta f_q W_N^{-pq}$.

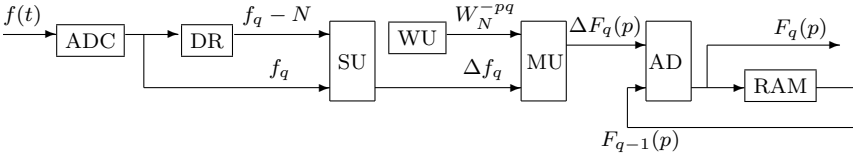


Figure 2. The spectrum analyzer functional diagram (4)

The spectrum analyzer functional diagram implementing algorithm (4) with matching is shown in Fig. 3, where CCU and AMU are the complex conjugate device and additional multiplier, respectively [10].

This algorithm has a remarkable property which permits one to organize SSA so that one complex multiplication may be used for computing two, four, and even eight (for complex signals) spectrum harmonics at once [3-6, 27]. This may be done by presenting algorithm (4) as follows:

$$F(p) = F_{q-1}(p) + \Delta F_q(p), \quad p \in \overline{0, P-1}, \quad q = 0, 1, 2, \dots, \quad (5)$$

$$\Delta F_q(p) = \frac{1}{N} \left(f(q) - F(q-N) \right) \exp \left(-j \frac{2\pi}{N} pq \right). \quad (6)$$

The spectrum increments $\Delta F_q(p)$ may be used not only for spectrum harmonic p , but also for the spectrum harmonics

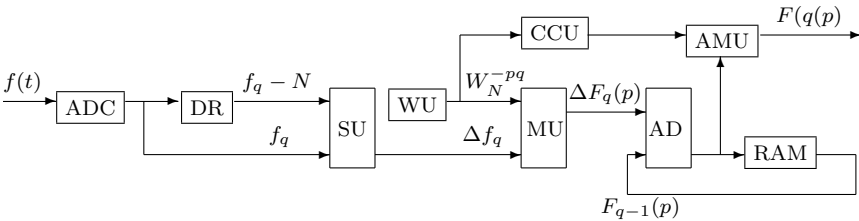


Figure 3. Spectrum analyzer function with matching

$$p_i = i \frac{N}{4} + p, i \in \overline{1, 3} \quad (7)$$

and

$$p_k = k \frac{N}{4} - p, k \in \overline{1, 4} \quad (8)$$

using known properties of the complex exponential function. A simplified summary of the algorithm (5) can be described as follows:

a) for harmonics of the form (7)

$$F_q(p_i) = F_{q-1}(p_i) + (-j)^{iq} \Delta F_q(p), \quad q = 0, 1, 2, \dots, \quad (9)$$

b) for harmonics of the form (8)

$$F_q(p_k) = f_{q-1}(p_k) + (-j)^k q \Delta F_q(-p), \quad q = 0, 1, 2, \dots, \quad (10)$$

where $F_q(-p)$ are complex conjugated spectrum increments $\Delta F_q(p)$, if the signal samples are real. If the signal samples are complex, the increments $\Delta F_q(-p)$ are generated by inverting the signs of the products of the signal increments $\Delta f_q = \frac{1}{N} (f(q) - f(q - N))$ with the imaginary part of the weighting function and then forming the appropriate algebraic sums.

Devices implementing algorithms (9) and (10) are presented in [28-32]. Their generalized functional diagram is shown in Fig. 4, where SW are switching components, m is the number of additional subranges, $m \in \{1, 3, 7\}$.

In the simplest case of complex signal analysis ($P = N$), the harmonic range is partitioned into two subranges $p \in \overline{0, \frac{N}{2} - 1}$, and by (7) for $i = 2$ we have $p_2 = \frac{N}{2} + p$. By (9) the readouts for the harmonics p_2 are determined by:

$$F_q(p_2) = F_{q-1}(p_2) + (-1)^q \Delta F_q(p). \quad (11)$$

i.e., the spectrum increment $\Delta F_q(p)$ for harmonic p obtained at the MU output are fed through SW to the input of the adder for subrange p_2 . In SW, as we see from (11), the sign of the increments $\Delta F_q(p)$ are either preserved or inverted, depending on the parity of the readout index q . To this end, it is sufficient to include in SW two operand sign control units (for the real and imaginary components of $\Delta F_q(p)$), which are controlled

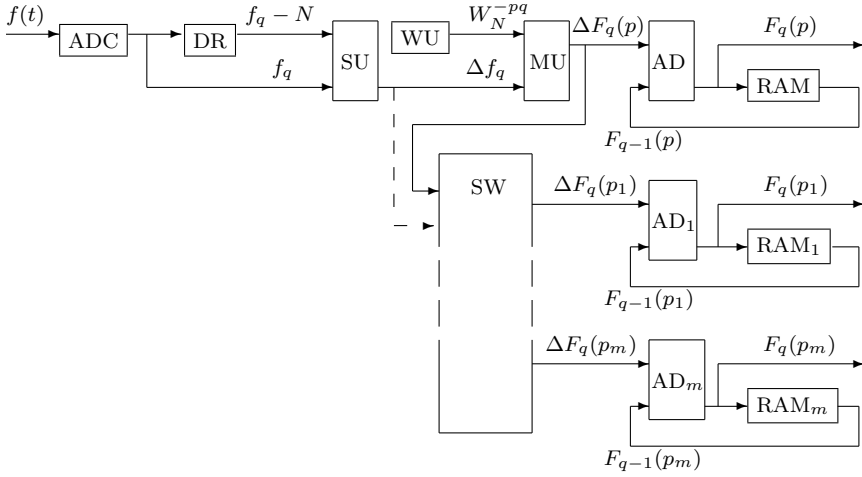


Figure 4. Generalized functional diagram: Implementation device

by the binary signal from the output of the least significant bit in the q -counter [28]. Operand sign control can be performed by exclusive OR circuits. Simultaneous evaluation of two harmonics doubles the functional speed at a small additional hardware cost.

Using (7) and dividing the harmonic range into four (for a complex signal) or two (for a real signal) subranges, we can similarly design appropriate analyzer structures [29, 30]. In this case SW, in addition to operand sign control unit, also contains multiplexers for the operations $(-j)^i q \Delta F_q(p)$. The activation sequence of these multiplexers with the corresponding operand sign control circuits can be determined from the data in Table I, which gives the dependences of the increments $\Delta F_q(p_1)$, $\Delta F_q(p_2)$, and $\Delta F_q(p_3)$ on $\Delta F_q(p)$ and q .

We see from Table I that the multiplexer control signal can be taken from the output of the least significant bit of the q -counter, while the control signals for the operand sign control circuits can be provided either by the outputs of the two least significant bits of this counter or by their logical XOR.

The SSA speed can be further doubled if, in addition to (7), we also use (8). In this case, SW must also include components that generate the spectrum increments $\Delta F_q(-p)$ (10). Moreover, the second SW input should receive from the SU the signal readout increment ΔF_q (dashed line in Fig. 4). This is needed for organizing simultaneous processing in corresponding subranges of operands of the harmonics $(2i+1)\frac{N}{8}$ and $k\frac{N}{4}$, $i \in \overline{1, 3}$, $k \in \overline{1, 4}$.

TABLE I. Multiplexer control signal

q mod 4	$\Delta F_q(p_1)$	$\Delta F_q(p_2)$	$\Delta F_q(p_3)$
0	$\Delta F_q(p)$	$\Delta F_q(p)$	$\Delta F_q(p)$
1	$Im\Delta F_q(p) - jRe\Delta F_q(p)$	$-\Delta F_q(p)$	$-Im\Delta F_q(p) + jRe\Delta F_q(p)$
2	$-\Delta F_q(p)$	$\Delta F_q(p)$	$-\Delta F_q(p)$
3	$-Im\Delta F_q(p) + jRe\Delta F_q(p)$	$-\Delta F_q(p)$	$Im\Delta F_q(p) - jRe\Delta F_q(p)$

To explain this technique, Tables II and III list the indices of simultaneously determined harmonics for real [31] and complex [32] signals. With the determination of the harmonic $p = \frac{N}{8}$, $\Delta F_q(p)$ is used to determine the harmonics $(2i + 1)\frac{N}{8}$, $i \in \overline{1, 3}$, and ΔF_q is used at the same time to calculate the harmonics $k\frac{N}{4}$, $k \in \overline{1, 4}$.

TABLE II. Indices of simultaneously determined harmonics

Subrange	Indices of simultaneously determined harmonics				
p	1	2	...	$\frac{N}{8} - 1$	$\frac{N}{8}$
p_1	$\frac{N}{4} - 1$	$\frac{N}{4} - 2$...	$\frac{N}{8} + 1$	$\frac{N}{4}$
p_2	$\frac{N}{4} + 1$	$\frac{N}{4} + 2$...	$3\frac{N}{8} - 1$	$3\frac{N}{8}$
p_3	$\frac{N}{2} - 1$	$\frac{N}{2} - 2$...	$3\frac{N}{8} + 1$	0

Structural diagrams of analyzers based on algorithms (9) and (10) are given in the invention descriptions [28-32]. These analyzers use modern hardware components and require some additional hardware costs to expand the frequency range by two, four, or eight times. The additional

hardware costs increase more slowly than the incremental benefits produced by group complex multiplication.

TABLE III. Indices of simultaneously determined harmonics

Subrange	Indices of simultaneously determined harmonics				
p	1	2	...	$\frac{N}{8} - 1$	$\frac{N}{8}$
p_1	$\frac{N}{4} - 1$	$\frac{N}{4} - 2$...	$\frac{N}{8} + 1$	$\frac{N}{4}$
p_2	$\frac{N}{4} + 1$	$\frac{N}{4} + 2$...	$3\frac{N}{8} - 1$	$3\frac{N}{8}$
p_3	$\frac{N}{2} - 1$	$\frac{N}{2} - 2$...	$3\frac{N}{8} + 1$	$\frac{N}{2}$
p_4	$\frac{N}{2} + 1$	$\frac{N}{2} + 2$...	$5\frac{N}{8} - 1$	$5\frac{N}{8}$
p_5	$3\frac{N}{4} - 1$	$3\frac{N}{4} - 2$...	$5\frac{N}{8} + 1$	$3\frac{N}{4}$
p_6	$3\frac{N}{4} + 1$	$3\frac{N}{4} + 2$...	$7\frac{N}{8} - 1$	$7\frac{N}{8}$
p_7	$N - 1$	$N - 2$...	$7\frac{N}{8} + 1$	0

4. Multichannel Matched Filtering on the Basis of SSA

SSA may be used for FIR-filtering [33] in invariant to information shift (stationary) mode. Such filters perform signal convolution with a given impulse response

$$s(\tau) = \int_{\tau-T}^{\tau} f^*(t)g(t-\tau)dt = \int_0^T f^*(t+\tau)g(t)dt. \quad (12)$$

In accordance with Parseval's theorem (12) is equivalent to the integral

$$s(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_{\tau}(\omega)G^*(\omega)d\omega, \quad (13)$$

where $G(\omega)$ is the unshifted impulse response spectrum, and $F_\tau(\omega)$ is the signal sliding spectrum in the analysis interval T . The discrete version of (13) is

$$r(q) = \sum_{p=0}^{P-1} F_q(p)G^*(p), \quad (14)$$

where $F_q(p)$ and $G^*(p)$ are samples of the spectra $F_\tau(\omega)$ and $G^*(\omega)$, P is the number of impulse response spectrum readouts. Expression (14) is equivalent to the traditional discrete correlation

$$r(q) = \sum_{k=0}^{N-1} f(k+q)g(k). \quad (15)$$

The expression (14) is the formal description of the FIR-filtering algorithm on the basis of SSA [22]. Practical realization of the filtration procedure while processing with a set of D different impulse responses may be performed by the device, whose functional diagram is shown in Fig. 5, where SSA is a harmonic sliding analyzer; MU_i are multipliers, $i \in 1, D$; $ROM_G^*_i$ are complex conjugated transfer function ROMs of the spectral readouts for the appropriate filter channels; and AA_i are accumulating adders of the pair products.

In such a device mutually overlapped input signal readouts series are used, with the help of SSA, to continuously in time complete the sliding spectrum readouts series $F_q(p)$, $q = 0, 1, 2, \dots$. These values are simultaneously placed in all the MU_i . The outputs of the MU_i are connected to second inputs of the $ROM_G^*_i$, which, synchronously with each p -th harmonic readout of the sliding spectrum, present respective p -th harmonic readouts of the complex conjugated transfer function spectrum. The products from the MU_i outputs are placed into the AA_i inputs. At the end of each sliding step, we form the sum (14) of the readouts $r_i(q)$.

As follows from Fig. 5, each channel of filtering consists of one accumulating adder, one $ROM_G^*_i$, and one multiplier connected to the common output of the SSA. The channel structure is uniform throughout. The difference is determined by the contents of the transfer function spectral readouts $ROM_G^*_i$.

The concrete performances of the appropriate devices for multichannel filtering on the basis of SSA are given in [20] for real signals and [21] for complex signals.

A criterion of effectiveness of the multichannel filter based on the SSA is the number of complex multiplications used for realization of one step of sliding on the set of D channels of filtering. In accordance with algorithm (14) and the structure of the complex multichannel filter the number α_s

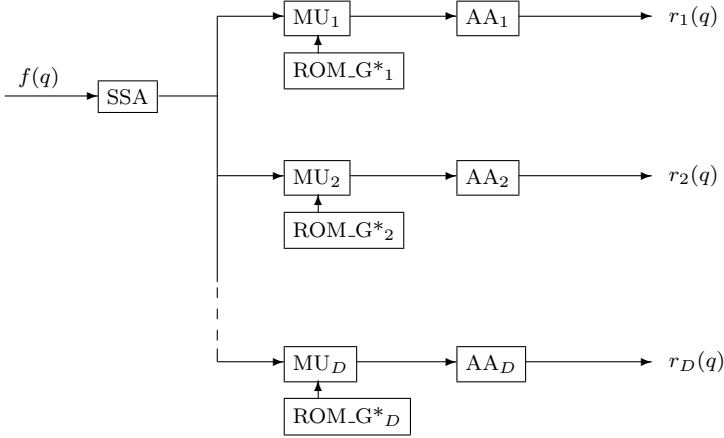


Figure 5. Varying impulse responses

of complex multiplication operations realizing one step of sliding is determined by the sum of the operations in the first stage of processing (stage of computing the sliding spectrum) and operations in the second stage of processing (D times of weighting and summation of the spectral components). Thus,

$$\alpha_s = P + DP = P(1 + D).$$

The appropriate computing expenditure realizing the standard correlation procedure of the complex multichannel filtering (15) is $\alpha_c = DN$.

Consequently, the benefit of the effectiveness of the multichannel filtering on the basis of SSA can be estimated by the ratio of the computing expenditures

$$B = \frac{\alpha_c}{\alpha_s} = \frac{DN}{P(1 + D)}. \quad (16)$$

The more N and the less P , compared with N , the more advantageous is filtering on the basis of SSA. It is possible to show that, in the case of matched (optimal) filtering, the number P practically coincides with the base of the useful signal $Q = \Delta FT$. The most benefit is realized in the use of matched filtering when $P = 1$. In this case the filter degenerates into D independent digital resonators. If $N \gg D$, then

$$B_{max} = \frac{DN}{1 + D} \approx N. \quad (17)$$

As the order of the realized filter or base of the filtered signal increases (P increases) the relative benefit decreases. In the limit, when $P = N$

$$B_{min} = \frac{D}{1+3} \approx 1, \quad (18)$$

and the effectiveness of the digital filter on the basis of SSA does not essentially differ from the effectiveness of the correlation filter.

It is important to note that the decrease in computing expenditures, when using SSA, leads to an almost proportional decrease in the hardware volume used. For example, when realizing D -channel filtering on the basis of correlation convolution (15) it is necessary to use D multipliers and adders working in parallel. The speed of each is determined by the necessity to perform N multiplications and additions for one sliding step. When realizing multichannel filters on the basis of SSA the number of multipliers and adders used to achieve the same productivity may be decreased approximately by N/P times, because each channel performs only P multiplications. In the extreme case when organizing D digital resonators ($P = 1$) it is sufficient to use solely the multiplier in the SSA, since the secondary processing in each channel completely degenerates and the SSA itself serves as the multichannel filter.

When filtering simple and low complexity signals in conditions of high degree a priori frequency uncertainty, when $N \gg P$, multichannel filter on the basis of SSA has advantages in hardware expenditures not only in comparison with correlation FIR-filtering but also with very effective filtering on the basis of FFT [33].

Multichannel filtering devices contain a direct Fourier transform unit. The output is connected with D filtering channels, each of which consists of a weighting unit and an inverse Fourier transform unit. These devices, without special buffer memory units, form the readouts of filtering the signal irregularly in time. This distorts the time scale and the regularity of the output information. To achieve stationary mode using these devices, we require additional hardware expenditures. Furthermore, the presence in each channel of an inverse Fourier transform unit increases weight, volume, and cost of the hardware. Devices [20, 21], which realize the discussed algorithm of filtration, are free of these drawbacks.

5. Recommendation on Spectrum Sliding Analysis Realization

It is also possible to increase the response speed of the instant digital spectrum analyzers using table multiplication [34]. This may be used if there is enough memory for appropriate tables.

An A-D converter with the aim to increase the accuracy may be used. A converter [35] with Sample-Hold circuit [36] allows an increase in both the accuracy and the speed. Project [37] showed that it is possible to achieve an accuracy of 16 bits with the sampling frequency up to 2 MHz. Projects [38] and [39] showed that using Xilinx PLAs Virtex-2 [40] it is possible to implement inventions [31] and [32] with the sampling frequency up to 3.9 MHz for the size of processing extracts $N = 256$ and the accuracy of 8 bits.

Spectrum digital analyzers output only Cartesian constituents of the spectrum complex samples. If only the modulus of these samples is required, it is possible to increase the response speed by using approximating methods of modulus calculations based on comparison, shift, and summation [41, 42].

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