FOUR PROBLEMS IN RADAR

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1. Introduction

Radar, short for RAdio Detection And Ranging, was invented almost a century ago by Christian Hulsmeyer in Düsseldorf, Germany. His patent for telemobiloskop, No. 165,546, issued 30 April 1904, was a collision prevention device for ships. However, Robert Watson-Watt is often given credit for inventing radar. Thirty years later, researchers were experimenting with radio transmission and reception at the Naval Research Laboratory in Washington DC. The experiment set up was such that the communications stations were on opposite sides of the Potomac River. Interference occurred each time a ship passed between these two communication stations, and proved to be a reliable indicator of an object present regardless of weather conditions. Since that time, great strides in radar have brought us air traffic control, airborne synthetic aperture radar for crop production assessment, and much more.

Radars operate via transmission through a directional antenna. Objects within the field of view scatter radio frequency (RF) energy in all directions, some towards the receiving antenna. Reflections from the clutter are undesired, and may mask signals from targets.

Throughout the twentieth century, great progress in and numerous applications of radar emerged as being commercially and militarily successful. Initially, investigations into electromagnetic theory supported experimentation with radio wires. Then, in the 1930's, interest in the Ultra High Frequency (UHF) (300 MHz) band resulted in the first surveillance and fire control systems. By the 1940's, microwave radars operating in the 500 MHz to 10 GHz band were developed. This resulted in long range search and track radars by the late 1950's.

Advances in waveform and signal processing technology ushered in a new era starting in the 1960's. The first airborne radars were developed. Phased array technology made low sidelobe antennas available for the Airborne Warning And Control System (AWACS) wide area surveillance platform. Sidelobe cancellers emerged as the first adaptive signal processing technology designed to mitigate electromagnetic interference and jammers. Generalization of the sidelobe canceller concept, in conjunction with multi-channel phased array technology, led to space-time adaptive processing algorithms, architectures, and system concepts by the 1980's. During this same time, space-based radar (SBR) emerged as technically feasible for wide area surveillance for theater wide surveillance. As the 20th century came to a close, advances in digital technology, computing architectures and software, and solid state radio frequency devices offered some of the most exciting opportunities for fielding new radars with previously unheard of capabilities. All of this leads to the four challenge problems in radar discussed below.

2. Radar Fundamentals

In Figure 1, a simplified block diagram presents the fundamental building blocks and inter-connectivity essential to the functioning of a radar. Here, a classical system using a reflector antenna is presented, while modern systems use phased arrays and multi-channel receivers.

In a radar system, the modulator generates a low power signal which drives the transmitter. In the transmitter, the signal may be converted in frequency from baseband, and is amplified, often to many kilowatts of peak power. The duplexer protects the receiver during transmit, and directs backscattered energy to the receiver. The antenna focuses transmit energy into narrow beams to localize in angle, as well as intercept returns from targets on receive. The receiver amplifies the return signal in order to overcome component noise, down converts the radar signal to a low intermediate frequency (1 MHz), match filters the radar returns, envelope detects the signal and digitizes it. The synchronizer is used for waveform generation, timing, and control of the transmit pulse, and measures range to the target on receive. The signal processor is designed to separate target returns from clutter and other interference, and estimate target parameters. The tracker further processes radar returns

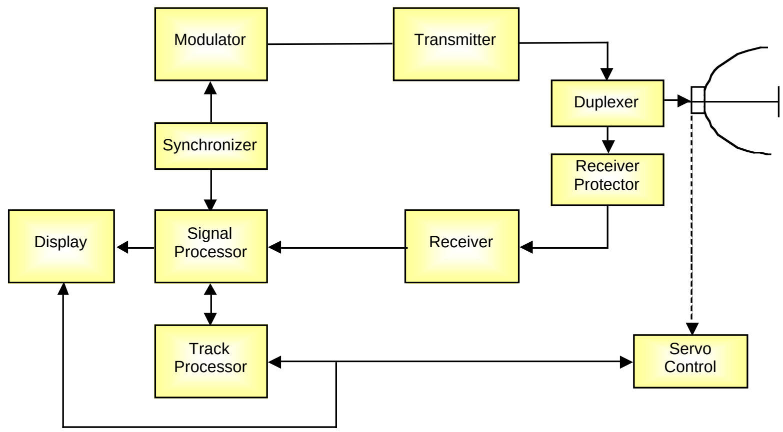

Figure 1. Basic Radar Architecture.

to present target history and predict future position for display to the operator.

The radar range equation is used to size a system to mission requirements. In its simplest form, the range equation starts with transmit power P_t, at a distance R from the radar with mainbeam transmit antenna gain G_t, the power density is

$$\frac{P_t G_t}{4\pi R^2}\,. \tag{1}$$

The power intercepted by a target of radar cross section σ and reradiated towards the radar is

$$\frac{P_t G_t \sigma}{4\pi R^2}\,. \tag{2}$$

The power density of the target return at the radar is

$$\frac{P_t G_t \sigma}{(4\pi R^2)^2}\,. \tag{3}$$

The received power is

$$\frac{P_t G_t \sigma A_r}{(4\pi R^2)^2}\,, \tag{4}$$

where

$$A_r = \frac{\lambda^2 G_r}{4\pi} \tag{5}$$

is the receive aperture at the radar. The result is

$$\frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4}. \tag{6}$$

The received power, P_r, can be written in terms of signal-to-noise ratio (SNR), S/N, and thermal noise power kT_0BN_F, where k is Boltzman's constant, T_0 is the noise temperature of the radar, B is the noise bandwidth of the radar receiver, and N_F is its noise figure. Substituting in to Equation (6), we get

$$P = (S/N)\, KT_0BN_F = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4}. \tag{7}$$

This equation must be satisfied to achieve detection performance commensurate with a given SNR. In addition to noise figure (N_F), numerous other losses, L, degrade the detection performance of a radar. They include hardware losses, atmospheric effects, and beam and filter shape losses, etc. The maximum range of a radar may then be computed using the formula

$$R_{\max}^4 = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 KT_0LBN_F\, (S/N)_{\text{req}}}. \tag{8}$$

In conventional radar, the output of the receiver is detection processed and compared to an adaptive threshold to determine target present (hypothesis H_1) or target absent (hypothesis H_0). From here, target declarations are handed off to the tracker for further analysis and handoff to the fighter/interceptor.

3. Radar Waveforms

In airborne early warning radar for wide area surveillance, a number of tradeoffs must be considered before waveform parameters are selected. Among them are continuous wave or gated-wave modes of operation. In gated-wave operation, the pulse width and the pulse repetition rate must be selected to be comparable with the target characteristics as well as the radar hardware available to the systems engineer. Since Fourier analysis is a standard tool available for radar signal analysis, the gated waveform should be composed of a number of pulses regularly repeated at a high enough rate to meet the Nyquist sampling criterion. However, too high of a repetition rate causes other problems, most notably, ambiguities in range. With range ambiguities, distant target returns may be masked by very strong close-in clutter. As such, the pulse repetition rate may have to be lowered. The tradeoffs in pulse duration, pulse repetition rate, and pulse shape impact radar resolution, accuracy, and ambiguities not just in range, but in Doppler as well. Doppler is important in radar because it is proportional to target radial velocity. Through velocity measurements, target position and engagement procedures are established.

Considering the tradeoff between low, medium, and high pulse repetition frequency (PRF), several issues must be discussed. Low PRF waveforms offer precise unambiguous measurements of range, and permit simple elevation sidelobe clutter rejection. This must be balanced with difficulties in resolving Doppler ambiguities, and suffer from poor performance against ground moving targets.

In high PRF systems, range ambiguities compound the clutter rejection problem, but the clutter free Doppler may be quite large. An appropriate compromise may be medium PRF, if low elevation sidelobes are easily incorporated into the systems designed. Also complicating the waveform design are pulse compression and modulation characteristics. The purpose of pulse compression is to permit high resolution modes of operation with long duration large bandwidth pulses that meet energy on target requirements established by the radar range equation. Waveform diversity enabled by the plethora of possibilities in waveform generation, timing and control, is now commonly discussed in the literature. Waveform diversity can encompass many aspects of the signal design problem, including frequency division multiplexing, pseudo-random phase coding, and pulse compression chirp rate diversity. Similarly, this concept can easily be expanded to encompass both temporal and spatial diversity. One simple concept for waveform diversity, called spatial denial, prohibits non-cooperative receivers from interfering with mainlobe signals by altering the nature of the sidelobe structure. This can be accomplished through the application of individual waveform generation, timing and control electronics at each channel in a phased array, or by augmenting existing equipment with auxiliary antennas (at least two) designed to accomplish the same mission. This is illustrated in Figure 2.

Spatial denial is achieved as illustrated in Figure 3.

The effects of sidelobe modulation of the interferometric spatial denial antennas are to produce a broad null along the end fire axis of the antenna. If the two bracketing antennas are orthogonal to the orientation of the radar antenna, then mainlobe radar emissions are unaffected by the spatial denial radiation, while the radar sidelobes are completely obscured by this same energy. The resultant effect is to prevent the non-cooperative receiver from sampling the radar emissions, and cohering to them. In Figure 4, we illustrate the radiated waveforms that emerge from a waveform diverse transmit aperture.

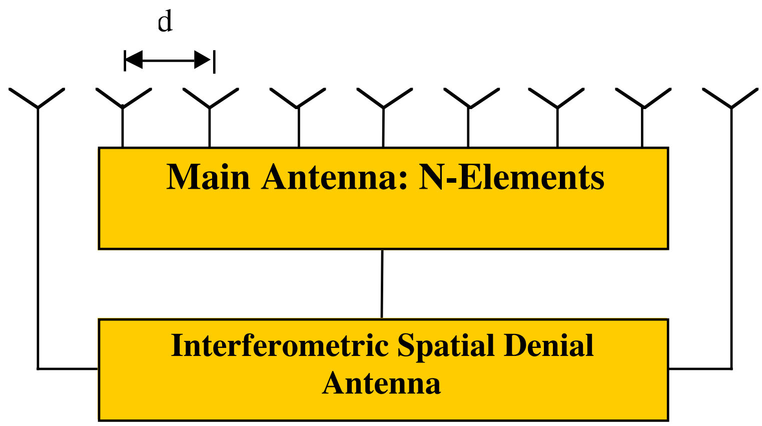

Figure 2. Interferometer Pattern.

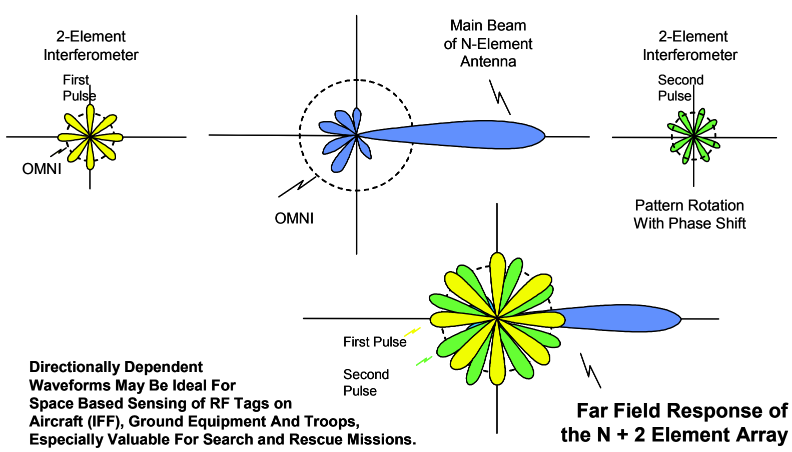

Figure 3. Spatial Denial Antenna Pattern.

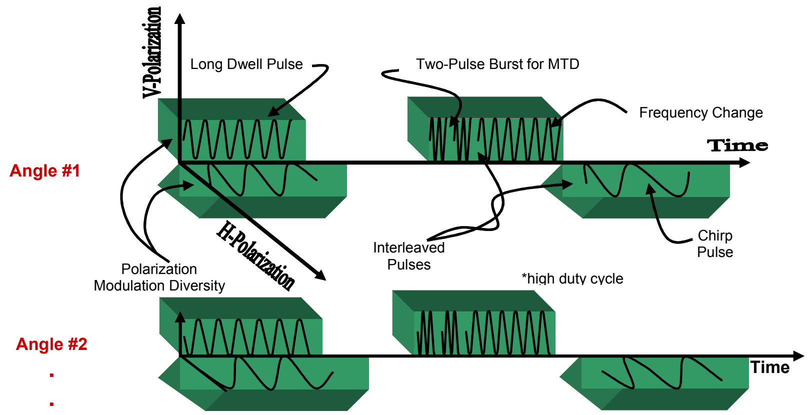

Figure 4. Transmit Waveform Diversity.

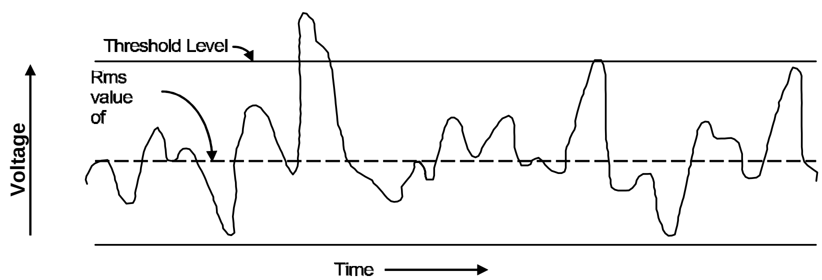

Figure 5. Typical Envelope of Radar Output.

Here, multi-mission, multi-mode waveforms are transmitted simultaneously in different directions, accomplishing different objectives, all without mutual interference.

4. Signal Processing

To achieve adequate detection in noise, the radar engineer must consider the minimum detectable signal given the radar receiver characteristics, the required SNR given the detection criteria and the radar range equation. The minimum detectable signal is determined by the ability of a receiver to detect a weak radar return as compared to the noise energy that occupies the same portion of the frequency band as the signal energy. This is illustrated in Figure 5.

The root mean-square (RMS) value of the in-band noise establishes the absolute limit to sensitivity exhibited by a radar receiver. Due to

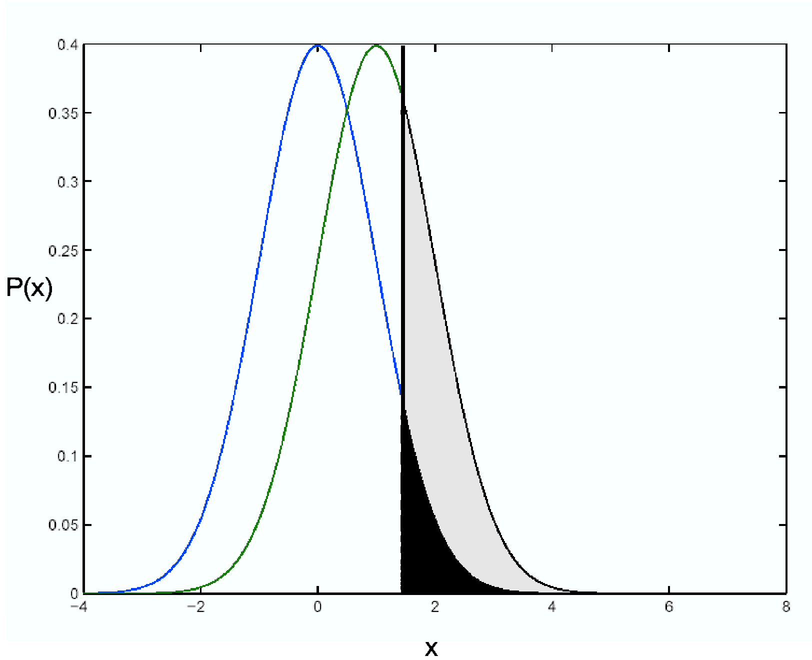

Figure 6. Effects of Target on Distribution Function.

the random nature of noise, the receiver threshold level is often many decibels (dB) above the RMS noise level. The presence of a target, often modeled as a constant signal, will alter the statistics of the received signal by shifting the mean value of the baseband voltage waveform. This shift in mean value is what permits detection processing using the various criteria that have emerged including Neyman-Pearson, Bayes, etc. Figure 6 illustrates the effect of a target on the distribution of a received signal.

One problem in signal processing is false alarms due to thermal noise fluctuations. The ultimate tradeoff is between detection probability, which is a function of the range equation and the threshold level of the receiver, and the false alarm rate, which is a function of the noise statistics and the threshold level. The common element is the threshold setting. The probability of detection for a sine wave in noise as a function of the signal-to-noise (power) ratio and the probability of false alarm are presented in Figure 7.

The false alarm rate for wide area surveillance radars must be small because there are so many range, angle, Doppler biases with the potential for false alarms. For a 1 MHz bandwidth, there are on the order of 10^6 noise pulses per second. Hence, the false alarm probability of any pulse

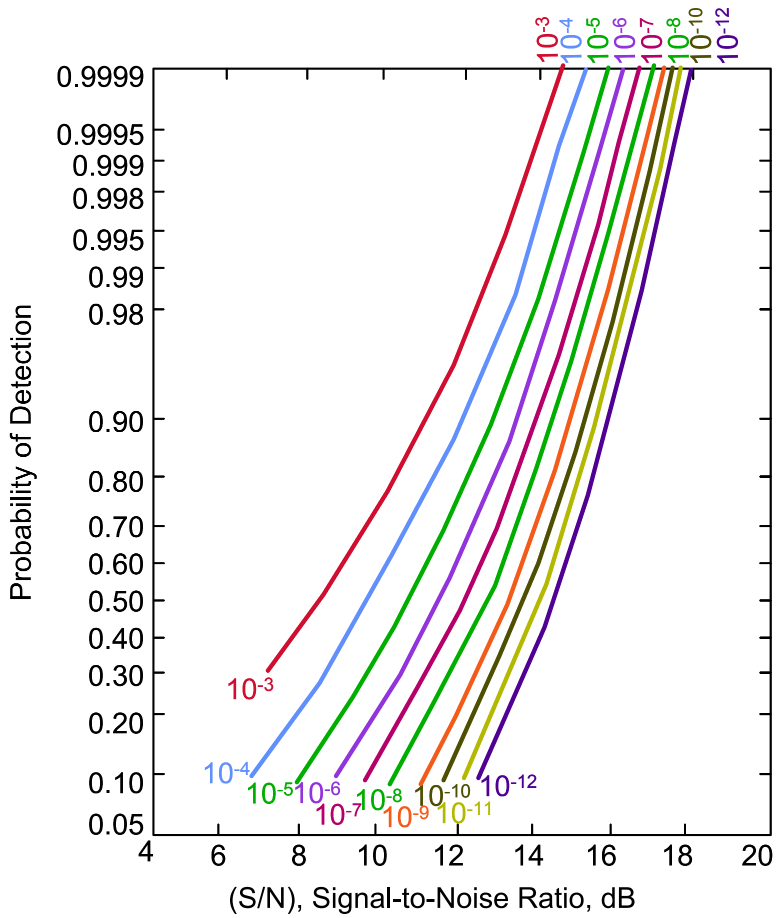

Figure 7. Probability of Detection vs. SNR.

must be small, less than one in a million. Non Gaussian clutter is more complicated and will further compound the false alarm control problem. In radar, we consider the classical hypothesis testing approach to be most appropriate. We have Type I errors, which are false alarms, and Type II errors, which are missed detections. In the Neyman-Pearson receiver, we fix the probability of a Type I error (P_I) and minimize the probability of a Type II error (P_{II}). We can consider several aspects of the detection problem with an ideal observer, by maximizing the total probability of a correct decision or minimizing the probability of error $P(e)$, given by

$$P(e) = P_N P_I + P_{S+N} P_{II} \tag{9}$$

where P_N is the a priori probability of noise only present, P_{S+} is the a priori probability of a signal plus noise crossing the threshold, and P_I, P_{II} were defined above.

In the sequential observer, we fix the probability of error a priori and allow for the integration gain to vary. Of course, in all of the previous discussions, it was assumed that the likelihood ratio test is used to analyze measured radar data.

If an event has actually occurred, the problem of forming the best estimate of the source of the event uses Bayes rule, which states

$$P\left({}^{S+N}/_{Y}\right) = \frac{P\left({}^{Y}/_{S+N}\right) P\left(S+N\right)}{P\left({}^{Y}/_{S+N}\right) P\left(S+N\right) + P\left({}^{Y}/_{N}\right) P\left(N\right)}\,. \tag{10}$$

In signal processing, clutter is defined as unwanted radar returns from ground, sea, precipitation, birds, etc. The clutter parameters of interest in signal processing algorithm development include amplitude distribution, spatial extent, temporal stability, radar parameter dependence, etc.

Clutter characteristics must be understood in order to optimally select radar waveform parameters, as well as signal processing algorithms designed to separate target returns from interference. Finally, the signal plus any residual clutter energy must be tested for target present/target absent. The basic assumption here is that adequate clutter rejection has been achieved such that statistical hypothesis testing against thermal noise is adequate. In realistic radar environments, that is seldom the case, and the detector circuit must be altered to adequately address the impact of clutter residue on detection performance. Clutter residue out of a simple two pulse canceller can be impacted by amplitude and phase errors in the radar system. Figure 8 illustrate these effects, where

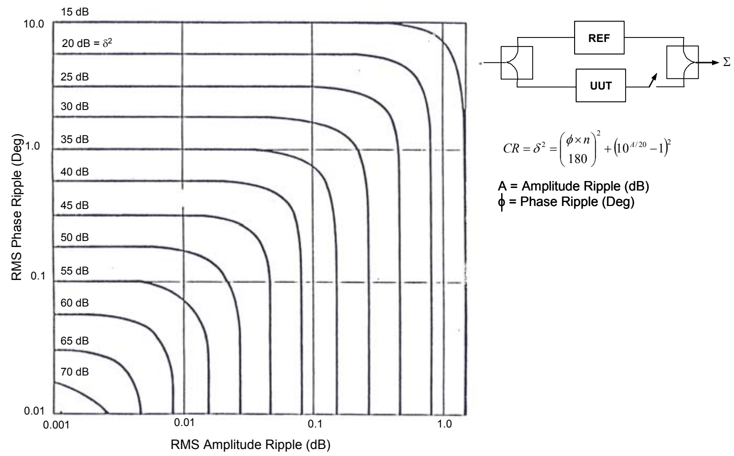

Figure 8. Cancellation Ratio

clutter residue (CR) is plotted as a function of RMS amplitude ripple (error) versus RMS phase ripple. As an example, an amplitude error of 0.1 dB limits the signal processor CR to less than 40 dB. If the interfering ground clutter is very strong, which it typically is in modern airborne radar, and exceeds the target return by more than 40 dB, then automatic detection processors will not be able to distinguish between threat targets and clutter residue, rendering the radar useless in this case.

5. Space-Time Adaptive Processing

In radar signal processing research, modern Space-Time Adaptive Processing (STAP) developments incorporate numerous disciplines, including applied statistics, linear algebra, software engineering, advanced computing technology, transmit receive module technology, waveform generation, timing and control, antennas, and system engineering. Each of these topics requires years of study to become an expert in any one area. This section focuses on a systems engineering approach to understanding the need for STAP in modern radar.

Consider an airborne wide area surveillance radar with a transmit mainlobe pointing broadside (perpendicular to the velocity vector of the airborne radar). In this example, an airborne threat target is approaching the radar in the general direction of the broad transmit mainbeam. Our goal is to separate the target return from ground clutter returns.

The spectral spread of ground clutter is governed by the equation

$$f_d = \frac{2v}{\lambda}\cos(\theta)\,. \tag{11}$$

In this equation, f_d is the Doppler offset of clutter arriving from the angle q, where q is measured with respect to the velocity vector of the airborne radar, v is the velocity of the radar platform, and λ is the radar wavelength and is proportional to the radar frequency f_c, through

$$\lambda = c/f\,, \tag{12}$$

where c is the speed of light. For a given angle θ_1, the clutter Doppler frequency is given by

$$f_{d_1} = \frac{2v}{\lambda}\cos(\theta_1)\,.$$

If a mainlobe target is approaching the radar at radial velocity , then the target Doppler is

$$f_{d_2} = \frac{2v_2}{\lambda}\,.$$

If $f_{d_2} = f_{d_1}$, then sidelobe clutter from angle θ_1 is going to compete with mainlobe target traveling with radial velocity v_2. If the sidelobe clutter is effectively a much larger scattering center than the mainlobe airborne target, then the detection process could easily produce a type II error and the threat target will go unreported. One approach to rejecting clutter competing with mainlobe targets is to lower the sidelobes of the antenna to the point where unwanted interference is suppressed to below the thermal noise floor of the receiver. Then only mainlobe clutter will remain, and these unwanted returns can be rejected by Doppler processing. However, this is unrealistic.

In order to produce extremely low sidelobes, an antenna aperture would be very large. Additionally, the antenna would have to operate in the far field of any large scattering center. In modern airborne radar, this is unrealistic.

If the radial velocity of the threat target were known a priori, then a receive antenna pattern and a platform velocity vector could be selected to place a null on the clutter that would compete with the target in the detection process. However, this is unrealistic, even under the simplest conditions. The only way to accomplish the task of detecting targets and rejecting interference is to adaptively place nulls in the sidelobe pattern to optimize performance. However, since angle-Doppler coupling ties sidelobe clutter position to clutter Doppler frequency offset, the adaptive nulling problem is not one-dimensional (1-D) (angle) but two-dimensional (2-D) (angle-Doppler).

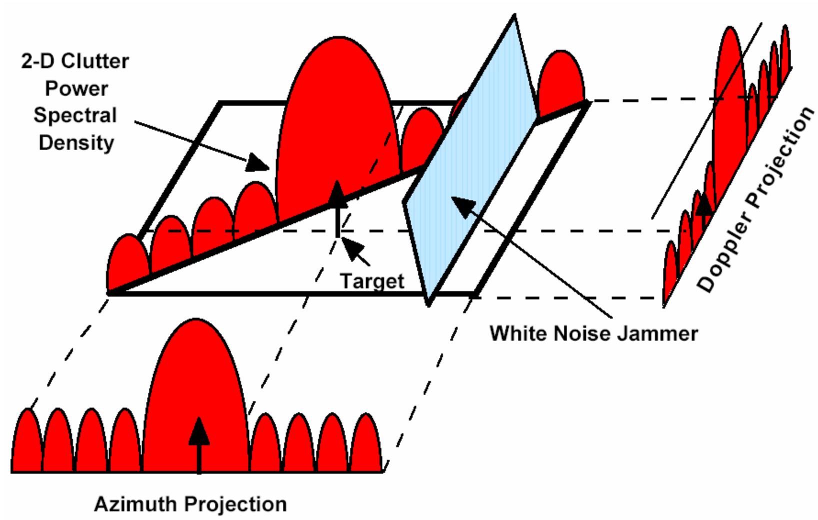

Figure 9. Angle-Doppler Profile.

In radar, angle and Doppler are derived variables. In a measurement system, data is collected in the space and time domains. It is important to note that space and angle are related by the Fourier transform, as are time and Doppler. This fact is exploited in STAP-based signal processing, and is illustrated in Figure 9.

As illustrated in Figure 9, mainlobe clutter and targets are widely separated in theory. In practice, the fundamental issue is leakage suppression among Doppler filters, or antenna beams. In space-time adaptive processing, the simplest approach to clutter rejection and target detection is through application of the sample matrix inversion (SMI) algorithm, which can be written as

$$lrt = \mathbf{s}^H \hat{\mathbf{R}}_k^{-1} \mathbf{x}_0 \,, \tag{13}$$

where s is the steering vector in angle, Doppler, or both, $\mathbf{x}_0$ is the measured data to be analyzed for target present $[H_1]$ or target absent $[H_0]$, and $\hat{\mathbf{R}}_k$ is the sample covariance matrix, where

$$\hat{\mathbf{R}}_k = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}_k \mathbf{x}_k^H \,. \tag{14}$$

The training data $\mathbf{x}_k$, $k = 1, 2, \ldots, K$, are selected from the immediate region where the data from the cell under test, $\mathbf{x}_0$, is collected. This helps to meet the independent, identically distributed assumption common in the literature. The measurement data, $\mathbf{x}_k$, are also assumed to be zero-mean complex Gaussian data vectors under the null hypothesis. In this simple test, if the sample covariance matrix is the identity matrix I, then the SMI test reduces to the discrete Fourier transform. However, $\hat{\mathbf{R}}_k$ will only be an identity matrix under the assumption that the clutter data vectors, $\mathbf{x}_k$, $k = 1, 2, \ldots, K$, are all white noise vectors, and K is large. More realistically, $\hat{\mathbf{R}}_k$ is not diagonal.

In order to simplify the processing in STAP based radar, the two-dimensional adaptive processing is decomposed into two one-dimensional processes. In radar, it is logical to perform Doppler filtering of the time-domain data from each channel in a STAP based radar first, and then apply the SMI algorithm in the spatial domain. As such, the adaptive degrees of freedom, i.e., the size of the steering vector s, is substantially reduced. Furthermore, the size of the sample covariance matrix is reduced as is the number of samples required to adequately estimate the covariance matrix.

In the 1980's, Kelly from MIT Lincoln Laboratory revisited the theoretical development that led to the SMI algorithm. The likelihood ratio test was developed under the Gaussian assumption where the data vectors were zero-mean under the null hypothesis. Additionally, the SMI algorithm was developed under the assumption of infinite iid training data, $K \to \infty$, for sample covariance matrix estimation, and that the sample covariance converges to the true covariance matrix as the number of samples tends towards infinity.

In Kelly's development, he applied one more condition to the likelihood ratio test, namely finite training data K. The generalization resulted in a new test statistic given by

$$lrt = \frac{\left|\mathbf{s}^H \hat{\mathbf{R}}^{-1} \mathbf{x}_0\right|^2}{\left(\mathbf{s}^H \hat{\mathbf{R}}^{-1} \mathbf{s}\right)\left(1 + \mathbf{x}^H \hat{\mathbf{R}}^{-1} \mathbf{x}\right)} . \tag{15}$$

Kelly's generalized likelihood ratio test exhibits an embedded Constant False Alarm Rate (CFAR) characteristic, meaning that the probability of type I error is fixed. Additionally, it is important to note that the presence of a T2-test in the denominator prohibits the separation of the filter function from the false alarm control function in this formula. As such, the separation of filter function from false alarm control is not possible. A further modification to the SMI algorithm is to incorporate the effects of angle-Doppler coupling (structure in the covariance matrix) into its development. This could be accomplished by conditioning in angle-Doppler coupling in the probability $P_{x/H_i}(x/H_i)$, $i = 0, 1$, used in the formulation of the likelihood ratio test. Once this is accomplished, the effects of angle-Doppler coupling on other statistical tests are easily established.

6. Four Problems in Radar

As analog hardware performance matures to a steady plateau, and Moore's Law provides for a predictable improvement in throughput and memory, it is only the advances in signal and data processing algorithms that offer potential for performance improvements in fielded sensor systems. However, it requires a revolution in system design and signal processing algorithms to dramatically alter the traditional architectures and concepts of operation. One important aspect of our current research emphasizes new and innovative sensors that are electrically small (on the order of 10 wavelengths or less), and operate in concert with a number of other electrically small sensor systems within a wide field of view (FOV). Our objective is to distribute the power and the aperture of the conventional wide area surveillance radar among a number of widely disbursed assets throughout the battlefield environment. Of course, we must have an algorithm for distributing those assets in real time as the dynamically changing surveillance demands. The mathematical challenge here relates to the traveling salesman problem. Classically, the traveling salesman must select his route judiciously in order to maximize potential sales. Recent analysis in the literature addresses multiples salesmen covering the same territory. This is analogous to our problem, where multiple unmanned aerial vehicle (UAV) based sensors are charged with the mission of detecting, tracking, and identifying all targets (friend or foe). Not only must these sensors detect and identify threat targets, they must also process data coherently across multiple platforms. Our mathematical challenge problem reduces to one in which the position and velocity all UAV-based sensors are selected to maximize detection performance and coverage area, and minimize revisit rate.

Enhancing one of the sensors described above, to be more like a classical radar with a large power-aperture product, leads to the second mathematical challenge problem to be addressed by this community. With a larger aperture and more precise estimates of target parameters (angle, Doppler), an opportunity to expand the hypothesis testing problem to include both detection and estimation emerges. Here, conventional wisdom dictates that we perform filtering and false alarm rate control as part of the detection process, yet perform track processing as a post-detection analysis, where the parameter estimation is focused upon target position and velocity history. Clearly, parameter estimation need not be accomplished as a post-detection process. Since this segmented approach to detection and track processing has been in effect for decades, it will require a dramatic demonstration of improvement before it will be embraced by the radar community.

A third challenge problem arises in the formulation of the Generalized Likelihood Ratio Test (GLRT). In Kelly's formulation of a GLRT, conditioning on finite sample support is incorporated into the basic test. As such, a statistical method developed under the assumption that only finite training data are available for sample covariance matrix formulation was made available. The next generalization to be made, in an extension of Kelly's GLRT, is to incorporate prior knowledge of the structure of the sample covariance matrix into the mathematical development of a statistical test. This mathematical structure arises due to the fact that the phase spectra of ground clutter as seen by an airborne radar is determined only by geometry, and remains independent of the underlying clutter statistics (except for initial phase). The effect of this geometric dependence is to localize the interference along a contour in the transform domain (Fourier analysis). Our objective is to formulate a single GLRT which incorporates the effects of finite training data as well as geometric dependence.

The fourth mathematical challenge facing the modern radar engineer is to incorporate adaptivity on transmit into the basic formulation of the signal processing algorithm. Since this is a new research topic, an opportunity exists to formulate the basic mathematical framework for fully adaptive radar on both transmit and receive. Further extensions arise by incorporating the above challenge problems into this analysis.

7. Conclusions

The rapid recent development in signal processing and waveform generation, timing, and control, has led to opportunities for fielding a vast array of new radar systems. Among these new sensor suites, we find that challenging problems remain unaddressed. Most notable is the incorporation of prior knowledge concerning the clutter environment into the likelihood ratio test. This work is expected to be a simple extension of Kelly's work in the 1980's.

Of course, extension of the thresholding process to a "post tracker" implementation offers further opportunities for performance enhancements and mathematical formulations. A further enhancement to radar is obtained through application of a wide variety of radar signals to a diverse number of radars, all operating in concert in a battlefield environment. This gives rise to a fourth opportunity in radar, that deals with the fielding and positioning of these radars, much like the traveling salesman problem.

All of this leads to new opportunities to exploit advances in applied mathematics, physics, electronics, and computer engineering to advance the state-of-the art in radar. While radar is nearing its 100-th year as a patented product, the prospects for advancement are as great as ever. Our defining moment in radar may have occurred in World War II; our impact on society is yet to occur, thanks to the mathematicians, scientists, and engineers who continue to advance the technology.