SEQUENTIAL DETECTION ESTIMATION AND NOISE CANCELATION

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Abstract. A noise canceling technique based on a model-based recursive processor is presented. Beginning with a canceling scheme using a reference source, it is shown how to obtain an estimate of the noise, which can then be incorporated in a recursive noise canceler. Once this is done, recursive detection and estimation schemes are developed. The approach is to model the nonstationary noise as an autoregressive process, which can then easily be placed into a state-space canonical form. This results in a Kalman-type recursive processor where the measurement is the noise and the reference is the source term. Once the noise model is in state-space form, it is combined with detection and estimation problems in a self-consistent structure. It is then shown how parameters of interest, such as a signal bearing or range, can be enhanced by including (augmenting) these parameters into the state vector, thereby jointly estimating them along with the recursive updating of the noise model.

1 Introduction

In underwater acoustic signal processing, own-ship noise is a major problem plaguing the detection, classification, localization and tracking problems [1, 2]. For example, this noise is a major contributer to towed array measurement uncertainties that can lead to large estimation errors in any form of signal processing problem aimed at extracting quiet target information. Many sonar processing approaches deal with this problem by relying on straightforward filtering techniques to remove these undesirable interferences, but at the cost of precious signal-to-noise ratio (SNR).

This chapter addresses the idea of noise cancelation by formulating it in terms of joint detection and estimation problems [3, 4]. Since such problems are, in general, nonstationary in character, the solution must be adaptive, and therefore the approach taken here is to cast it as a recursive model-based problem. By model-based [5] we mean that as much *a priori* information as possible is included in the form of physical models, where the parameters of these models can be adaptively estimated. Such an approach leads naturally to state-space based recursive structures. In other words, we will be dealing with Kalman filter based processors.

We begin in the next section with the development of a recursive detection structure. This is followed in sections 3 & 4 with the development of the ship noise and signal models, respectively. Section 5 develops the detection/noise-cancelation scheme. Section 6 continues with the development by showing how this structure can provide enhanced estimation. Finally, Section 7 contains a discussion.

2 Sequential Detection

The detector is configured as a binary hypothesis test based on the Neyman-Pearson criterion. Thus, the optimum solution is given by the likelihood ratio, *i.e.*,

$$L(t) = \frac{Pr(\mathbf{P}_t|H_1)}{Pr(\mathbf{P}_t|H_0)}, \tag{1}$$

where $\mathbf{P}_t$ is the set of time samples of the spatial vector of pressure measurements, *i.e.*,

$$\mathbf{P}_t = \{\mathbf{p}(1),\ \mathbf{p}(2), \cdots, \mathbf{p}(t)\} = \{\mathbf{P}_{t-1}, \mathbf{p}(t)\}, \tag{2}$$

where H_1 and H_0 respectively refer to the signal present and the null hypotheses, and $\mathbf{p}(t)$ is the spatial vector of measurements at time t. Thus, using Bayes' rule, the probabilities in Equation 1 can be written as

$$Pr(\mathbf{P}_t|H_{1,0}) = Pr(\mathbf{p}(t)|\mathbf{P}_{t-1}; H_{1,0})Pr(\mathbf{P}_{t-1}|H_{1,0}). \tag{3}$$

Substitution into Equation 1 yields

$$L(t) = \frac{Pr(\mathbf{p}(t)|\mathbf{P}_{t-1}; H_1)}{Pr(\mathbf{p}(t)|\mathbf{P}_{t-1}; H_0)}\left[\frac{Pr(\mathbf{P}_{t-1}|H_1)}{Pr(\mathbf{P}_{t-1}|H_0)}\right], \tag{4}$$

but from Equation 1 we see that this can be written as

$$L(t) = L(t-1)\frac{Pr(\mathbf{p}(t)|\mathbf{P}_{t-1};H_1)}{Pr(\mathbf{p}(t)|\mathbf{P}_{t-1};H_0)}. \tag{5}$$

Upon taking logarithms, we can now write the *sequential log-likelihood* ratio as

$$\Lambda(t) = \Lambda(t-1) + lnPr(\mathbf{p}(t)|\mathbf{P}_{t-1};H_1) - lnPr(\mathbf{p}(t)|\mathbf{P}_{t-1};H_0), \tag{6}$$

which leads to the binary test implemented as

$$\begin{array}{rrll} & \Lambda(t) & \geq lnT_1 & \text{Accept H}_1 \\ lnT_0 < & \Lambda(t) & < lnT_1 & \text{Continue} \\ & \Lambda(t) & \leq lnT_0 & \text{Accept H}_0 \end{array} \tag{7}$$

Following Wald [6] these thresholds are usually chosen to be

$$T_0 = \frac{Pr_D}{Pr_{FA}} = \frac{1-Pr_{miss}}{Pr_{FA}}, \tag{8}$$

and

$$T_1 = \frac{1-Pr_D}{1-Pr_{FA}} = \frac{Pr_{miss}}{1-Pr_{FA}}. \tag{9}$$

In these thresholds, Pr_D and P_{FA} are probabilities of detection and false alarm chosen by the user.

Before we can continue with the detection problem, we must develop expressions for the relevant probability densities in Equation 6. In order to do this we must develop the models for the ship noise and the signal.

3 Ship Noise Model

It can be shown [5, 8, 9] that the optimum noise canceling structure can be formulated as an identification problem with the noise reference input $z(t)$ related to the ship noise estimate $\hat{\eta}(t)$ by a coloring filter $h(t)$, *i.e.*,

$$\hat{\eta}(t) = h(t)*z(t). \tag{10}$$

Using a canonical form, Equation 10 can be written in Gauss-Markov [10] form as

$$\eta(t) = \sum_{i=1}^{N_h} h_i z(t-i), \tag{11}$$

with $z(t)$ being the reference. This leads to the state equation

$$\xi(t) = A_\xi \xi(t-1) + B_\xi z(t-1) + w_\xi, \tag{12}$$

and the measurement equation

$$\eta(t) = C_\xi \xi(t) + v_\xi, \tag{13}$$

where

$$A_\xi = \begin{bmatrix} 0 & 0 \cdots & 0 \\ & & \vdots \\ & I_{N_h-1} & 0 \\ & & 0 \end{bmatrix}$$

$$B_\xi = [1 \;\; 0 \;\; \cdots \;\; 0]. \tag{14}$$

and

$$C_\xi = [h_1 \;\; h_2 \;\; \cdots \;\; h_{N_h-1} \;\; h_{N_h}]. \tag{15}$$

The recursive solution is given by the following algorithm.

$$\begin{aligned}
\xi(t|t-1) &= A_\xi \xi(t-1|t-1) + B_\xi z(t-1) && \text{[Prediction]} \\
\eta(t|t-1) &= C_\xi \xi(t-1|t-1) && \text{[Meas Pred]} \\
\epsilon_\eta(t) &= \eta(t) - \eta(t|t-1) && \text{[Innovation]} \\
R_{\epsilon_\eta \epsilon_\eta}(t|t-1) &= C_\xi \tilde{P}_{\xi\xi}(t|t-1) C'_\xi + R_{\nu_\xi \nu_\xi} && \text{[In Cov]} \\
\xi(t|t) &= \xi(t|t-1) + K_\xi(t) \epsilon_\eta(t) && \text{[Correction]} \\
K_\xi(t) &= C_\xi(t) \tilde{P}_{\xi\xi}(t|t-1) R^{-1}_{\epsilon_\eta \epsilon_\eta}(t|t-1) && \text{[Gain]}
\end{aligned} \tag{16}$$

The $\tilde{P}_{\xi\xi}(t|t-1)$ term is the state error covariance, and is computed as an integral part of the Kalman algorithm. The notation in these equations has been generalized in order to explicitly indicate the predictive nature of the algorithm. That is, $\xi(t|t-1)$ is the value of ξ at time t, based on the data up to time $t-1$.

4 Signal Model

The measurement system is taken to be a towed array of N receiver elements. The received pressure field at the array, as a function of space and time, is given by

$$p(x_n(t), t) = s(r_n(t), t) + \eta(x_n(t), t) + \nu(x_n(t), t), \qquad (17)$$

where x_n is the spatial location of the n^{th} receiver element on the array, s is the source (target) signal, η is the interfering own ship noise and ν is the ambient noise. It is assumed that the signal is a weak narrow-band planar wave given by

$$s(x_n, t) = a_0 e^{i(\omega_0 t - k[x_n(0)+vt] sin\theta)}. \qquad (18)$$

The radian frequency at the source is ω_0, θ is the bearing of the source and v is the speed of forward motion of the array. $k = \omega_0/c$ is the wavenumber and c is the speed of sound. Note that for this signal model, $x(t) = x(0) + vt$. That is, the array motion is modeled as a Galilean transformation on the receiver elements. The significance of this is that, in bearing estimation, the motion contributes information that is contained in the Doppler that enhances the estimate [11, 7]. The broadband measurement noise is modelled as zero-mean, white Gaussian. Note that we are not restricting the statistics to be stationary, so we can accommodate the nonstationarities that occur naturally in the ocean environment.

5 Joint Detection/Noise Cancelation

We now have everything we need to construct our processor. Under the null hypothesis, we have that $Pr(\mathbf{p}(t)|\mathbf{P}_{t-1}; H_0)$ is conditionally Gaussian, since η is a consequence of the Gauss-Markov model of Equations 16. Thus

$$Pr(\mathbf{p}(t)|\mathbf{P}_{t-1}; H_0) \sim N(\hat{\mathbf{p}}(t|t-1), R_{\epsilon_p \epsilon_p}(t|t-1)). \qquad (19)$$

Here, $\hat{\mathbf{p}}(t|t-1) = E\{\mathbf{p}(t)|\mathbf{P}_{t-1}; H_0\}$ is the conditional mean estimate at time t based on the data up to time $t-1$, and $R_{\epsilon_p \epsilon_p}$ is the innovations covariance, both of which are available from the estimator of Equations 16. Including the ship noise term in the signal model, we see that for the null hypothesis,

$$\mathbf{p}_o(t) = \eta(t) + \nu(t), \tag{20}$$

so that

$$\epsilon_{\mathbf{p}_o}(t) = \mathbf{p}(t) - \mathbf{p}(t|t-1) = [\eta(t) - \eta(t|t-1)] + \nu(t), \tag{21}$$

or

$$\epsilon_{\mathbf{p}_o}(t) = \epsilon_\eta(t) + \nu(t), \tag{22}$$

which has the corresponding covariance

$$R_{\epsilon_{\mathbf{p}_o}\epsilon_{\mathbf{p}_o}}(t) = R_{\eta\eta}(t) + R_{\nu\nu}(t). \tag{23}$$

In the case of a deterministic signal, it now follows that for the alternate hypothesis

$$\mathbf{p}_1(t) = \mathbf{s}(t) + \eta(t) + \nu(t). \tag{24}$$

From Equations 20 and 24 then, it now follows that the noise canceled log likelihoods are

$$lnPr(\mathbf{p}(t)|\mathbf{P}_{t-1}; H_0) = [\mathbf{p}_0(t) - \hat{\eta}(t)]' R^{-1}_{\epsilon_{p_o}\epsilon_{p_o}}(t)[\mathbf{p}_0(t) - \hat{\eta}(t)] \tag{25}$$

and

$$lnPr(\mathbf{p}(t)|\mathbf{P}_{t-1}; H_1) = [\mathbf{p}_1(t) - \eta(\hat{\mathbf{t}})]' R^{-1}_{\epsilon_{p_o}\epsilon_{p_o}}(t)[\mathbf{p}_1(t) - \hat{\eta}(t)], \tag{26}$$

where we have used the fact that

$$R_{\epsilon_{p_1}\epsilon_{p_1}}(t) = R_{\epsilon_{p_o}\epsilon_{p_o}}(t). \tag{27}$$

Finally, upon substitution of Equations 25 and 26 into Equation 6, the recursive detection/cancelation processor is given by

$$\begin{aligned}\Lambda(t) &= \Lambda(t-1) + [\mathbf{p}_1(t) - \eta(\hat{\mathbf{t}})]' R^{-1}_{\epsilon_{p_o}\epsilon_{p_o}}(t)[\mathbf{p}_1(t) - \hat{\eta}(t)] \\ &\quad -[\mathbf{p}_0(t) - \eta(\hat{\mathbf{t}})]' R^{-1}_{\epsilon_{p_o}\epsilon_{p_o}}(t)[\mathbf{p}_0(t) - \hat{\eta}(t)].\end{aligned} \tag{28}$$

6 Joint Estimation/Noise Cancelation

Consider a moving towed array of N elements. The signal at the n^{th} element is given by Equation 18. It has been shown that by including the motion in this manner, the variance on the bearing estimate is reduced, as compared to that of the conventional (beamformer) estimator [11].

Here, we wish to further enhance the estimation results by incorporating the noise canceler into a recursive estimation scheme. As can be seen in Equation 18, there are three parameters in the signal model. These are the amplitude, source frequency and bearing. However, if we choose to work in the phase domain, the amplitude is eliminated and we are left with two parameters, ω_0 and θ. Including the ship noise, Equation 18 generalizes to[1]

$$s(x_n, t) = a_0 e^{i(\omega_0 t - k[x_n(0)+vt] sin\theta)} + \eta(t). \tag{29}$$

The hydrophone measurement is then

$$p_n = a_0 e^{i(\omega_0 t - k[x_n(0)+vt] sin\theta)} + \eta(t) + \mathrm{v}(t), \tag{30}$$

so that the canceled measurement for the Kalman filter is

$$p_n - \hat{\eta}(t) = a_0 e^{i(\omega_0 t - k[x_n(0)+vt] sin\theta)} + \mathrm{v}(t). \tag{31}$$

Although the source frequency appears to be a nuisance parameter, its inclusion is necessary in order to obtain the performance improvement by including the bearing information contained in the Doppler. This is sometimes referred to as the *passive synthetic aperture effect*. Making the assumption that the bearing changes slowly in time, the recursive estimator can now be cast in the form of a Kalman filter with a "random walk" state equation [11]. That is

$$\Theta(t|t) = \begin{bmatrix} \theta(t|t) \\ \omega_o(t|t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta(t|t-1) \\ \omega_o(t|t-1) \end{bmatrix}. \tag{32}$$

Since we wish to work in the phase domain, the measurement is based on the exponent of Equation 18, and is given by

$$y_n(\Theta) = (\omega_0/c)([x_n - x_{n-1}] + vt) sin\theta \quad n = 1, 2, \ \cdots, \ N-1. \tag{33}$$

[1] Although this development is based on a narrow band signal model, it can be easily generalized to accommodate a broadband model

Note that the $\omega_0 t$ term does not appear as it does in Equation 18. This is due to the fact that it is only the phase *differences* that are relevant to the problem; thus, there are $N-1$ phase difference measurements for N hydrophones. There is an auxiliary measurement equation that is based on the observed frequency. This is basically the Doppler relation and is given by

$$y_N(\Theta) = \omega = \omega_0(1 + (v/c)sin\theta), \tag{34}$$

with ω being the observed radian frequency. Note that the measurement equations are nonlinear due to the appearance of the term $\omega_0 \sin\theta$. Thus, we will need to use the extended Kalman filter.

The joint parametrically adaptive model-based processor (Enhancer/Estimator) in now given by

$$\begin{aligned}
\xi(t|t-1) &= A_\xi \xi(t-1|t-1) + B_\xi z(t-1) && \text{[Prediction]} \\
\Theta(t|t-1) &= \Theta(t-1|t-1) && \\
\eta(t|t-1) &= C_\xi \xi(t-1|t-1) && \text{[Meas Pred]} \\
\mathbf{y}(t|t-1) &= c[\Theta(t-1|t-1)] && \\
\epsilon_\eta(t) &= \eta(t) - \eta(t|t-1) && \text{[Innovation]} \\
\epsilon_{\mathbf{y}}(t) &= \mathbf{y}(t) - \mathbf{y}(t|t-1) - \eta(t|t-1) && \\
R_{\epsilon_\eta \epsilon_\eta}(t|t-1) &= C_\xi \tilde{P}_{\xi\xi}(t|t-1) C_\xi' + R_{\nu\nu}(t) && \text{[In Cov]} \\
R_{\epsilon_{\mathbf{y}} \epsilon_{\mathbf{y}}}(t|t-1) &= J_\Theta(t) \tilde{P}_{\Theta\Theta}(t|t-1) J_\Theta'(t) + R_{\mathrm{vv}}(t) && \\
\xi(t|t) &= \xi(t|t-1) + K_\xi(t)\epsilon_\eta(t) && \text{[Correction]} \\
\Theta(t|t) &= \Theta(t|t-1) + K_\Theta(t)\epsilon_{\mathbf{y}}(t) && \\
K_\xi(t) &= \tilde{P}_{\xi\xi}(t|t-1) C_\xi R^{-1}_{\epsilon_\eta \epsilon_\eta}(t) && \text{[Gain]} \\
K_\Theta(t) &= \tilde{P}_{\Theta\Theta}(t|t-1) J_\Theta(t) R^{-1}_{\epsilon_{\mathbf{y}} \epsilon_{\mathbf{y}}}(t) &&
\end{aligned} \tag{35}$$

The term $J_\Theta(t)$ in the last of Equations 35 is the Jacobian of the (nonlinear) measurement of Equations 33 and 34, which are collectively written as $c[\Theta]$ in the third of Equations 35, *i.e.*,

$$J_\Theta = \frac{\partial c[\Theta]}{\partial \Theta}. \tag{36}$$

That is, the nonlinearity of the measurement equations necessitates the use of the EKF [5], which in turn, requires the Jacobian.

7 Discussion

We have shown theoretically that recursive model-based processing can provide adaptive processing schemes that are capable of enhancing both detection and estimation procedures through the proper application of physical models. Here, we have used an all-pole model of own-ship noise and a recursive detector to enhance the detection of a signal. Further, we have shown how the use of such noise models, along with a realistic signal model, can enhance towed-array bearing estimation.

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